Izabela Pruchnicka-Grabias

DYNAMIC HEDGING STRATEGIES APPLIED BY BANKS FOR SHORT POSITIONS IN CURRENCY OPTIONS

Introduction

The majority of options sold by banks operating on the Polish financial market are mainly currency options. The most popular currency pairs are: EUR/USD, USD/PLN, and EUR/PLN. For speculation sometimes options on USD/YEN are used. Options on such currency rates as EUR/YEN, GBP/USD or GBP/EUR are less important for the market. Options on Swiss Franc, Canadian Dollar, Austrian Shilling or Scandinavian currencies are sold only for a special demand of a customer. That is why the author chooses for the analysis hedging strategies for currency options on USD/PLN.

On the Polish market there are two ways of hedging short option positions by banks, that is taking the opposite position in a similar option or a so called delta-hedging which means buying the appropriate amount of the underlying asset. The first one is the example of the static hedging, so it will not be discussed in the paper. The most popular way of hedging short option positions is delta-hedging. It was proposed by Fisher Black and Myron Scholes. It is based on the construction of the risk-free portfolio that consists of an option and the proper amount of an underlying asset that will compensate for losses generated on the option.

Delta and Factors Influencing It

Generally speaking, option positions are difficult to hedge because of many factors influencing the value of these instruments. These are: the underlying asset value, volatility of the underlying instrument, risk-free interest rate, and the exercise price of an option as well as time to maturity. The influence of the above mentioned factors on the option price is described by so called Greek letters and it is non linear, which makes risk management to be especially complex. In this paper it is delta to be the base for the hedging strategy. It is defined as option's price sensitivity to the underlying asset changes:[1]

$$\Delta = \frac{\partial V}{\partial S}$$

For a call option it can be calculated as:[2]

$$\Delta = N(d_1)$$

whereas for a put option:

$$\Delta = N(d_1)-1$$

Where:
$N(d_1)$ – the cumulative normal distribution function for d_1 defined as:[3]

$$d_1 = \frac{\ln\left(\frac{S}{X}\right) + \left(r + \frac{\sigma^2}{2}\right)T}{\sigma\sqrt{T}} \qquad (1)$$

where:
σ – volatility of an underlying asset,
X – exercise price of an option,
V – option's price,
r – risk-free interest rate,
T – time to maturity.

1 F. Black, M. Scholes, *The pricing of options and corporate liabilities*, Journal of Political Economy 1973, No. 87, May/June, 1973, p. 637–659.
2 Ibidem.
3 Ibidem.

Delta must be monitored for the whole hedging period and changes in the risk-free portfolio must be made, which is both time and costs-consuming. In the Black-Scholes environment there are no transaction costs, which makes the delta-hedging less complex.

In the world where transaction costs exist, frequent trading may make a hedging strategy to be more expensive than real advantages that derive from it. As H. E. Leland[4] emphasizes, transactions costs associated with replicating strategies are path-dependent: they depend not only on the initial and final stock prices, but also on the entire sequence of stock prices in between. Computation of the maximum transaction costs is a nontrivial problem. Rare trading can reduce costs, however it maximizes the hedging error.

The hedging error, defined as the difference between the return to the portfolio value and the return to the riskless asset over the rebalancing (rehedging) interval, depends on the length of the interval. When rebalancing intervals are relatively small and thus trading takes place very frequently, then the expected hedging error may be relatively small[5].

The Range of Examinations

In the paper the author assumed that transaction costs exist (contrary to the Black and Scholes assumptions) because in fact they play an important role on the financial market, so not taking them into consideration may lead to wrong conclusions. The point of the paper is to show to what extent they can be reduced thanks to different modifications of the delta-hedging strategy. The paper deals with hedging in the presence of transaction costs for the following hedging strategies:

- Every day delta-hedging,
- Delta hedging at fixed period of time (every 2, 3, 5, 10 days),
- The delta tolerance strategy,
- Hedging to a fixed bandwidth,
- The underlying asset tolerance strategy.

The examinations were made for the put currency option on USD/PLN. Currency rates between 01.10.2007 and 28.12.2007 were taken into consideration. Examinations were made under the following assumptions:

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4 H. E. Leland, *Option Pricing and Replication with Transactions Costs*, The Journal of Finance, vol. XL, No. 5, December 1985, p. 1284.

5 M. Mastinsek, *Discrete-time delta hedging and the Black-Scholes model with transaction costs*, Mathematical Methods of Operations Research, nr 64, 2006, p. 227–228.

- ❖ Domestic risk free interest rate is 5%;
- ❖ One option contract refers to 100 000USD;
- ❖ Foreign risk free interest rate is 3.75%;
- ❖ Standard deviation of the USD/PLN rate is 51% (counted by the author for a six month period between April and September 2007). It is assumed that volatility is constant during option's life;
- ❖ Exercise price of the option is 2,2500;
- ❖ Transaction costs are equal to 0.3% and they are proportional, which means that there is no minimum or maximum value;
- ❖ Option's time to maturity is 90 days;
- ❖ Stock prices' rates of return are normally distributed;
- ❖ Stock prices follow the random walk:

$$dX(t) = \mu X dt + \sigma X dB(t),$$

provided that:

$$\mu(X,t) = \mu X \text{ and } \sigma(X,t) = \sigma X$$

where:
μ – the constant rate of the underlying asset's price increase,
σ – asset price volatility,
B – Brownian motion,
X – random variable,
t – time.

The Black Scholes Continuous Delta Hedging vs Delta Hedging at Fixed Period of Time

As mentioned above, Black and Scholes assumed that transaction costs are zero. Under such assumptions rehedging to a perfect position can be done continuously. In the real world where transaction costs exist and what's more they play a significant role (especially in emerging markets where they are the highest and besides the lack of liquidity lowers the efficiency of the discussed dynamic hedging strategies), different methods of reducing costs of hedging can be applied. The simplest one is to change the hedge position not continuously but once a day. Such an every day delta hedge lets keep costs much lower than the continuous hedging. Costs can be reduced even more if a hedger rebalances his portfolio every few days. It can be done for instance every 2, 3, 5 or 10 days. The number of days depends for example on the risk tolerance of a hedger. The higher his risk aversion is the more frequent changes in a portfolio must be made. However, it is not only the risk tolerance to be taken into consideration. Other economic factors as for instance the volatility of the underlying instrument are also important. The higher it is the more frequent changes should be made.

Tables 1, 2 and 3 depict currency rates[6] used for calculations, as well as data calculated by the author, i.e. delta and its one day changes, as well as costs of every day hedging for the examined options.

Table 1. Costs of delta-hedging in October 2007 for every day changes in a zero-risk portfolio

Quotation date	USD/PLN currency rate (average)	Delta	Delta change during one day	Costs of delta-hedging for one put option on 100000 USD (in USD)
10.01.2007	2,6445	–0,21665	–	64,99
10.02.2007	2,6630	–0,20869	0,007961	2,39
10.03.2007	2,6646	–0,20801	0,000678	0,20
10.04.2007	2,6656	–0,20759	0,000423	0,13
10.05.2007	2,6563	–0,21155	–0,00396	1,1871
10.08.2007	2,6662	–0,20734	0,00421	1,26
10.09.2007	2,6575	–0,21103	–0,0037	1,11
10.10.2007	2,6476	–0,2153	–0,00427	1,28
10.11.2007	2,6204	–0,22737	–0,01207	3,62
10.12.2007	2,6281	–0,2239	0,003466	1,04
10.15.2007	2,6133	–0,2306	–0,0067	2,01
10.16.2007	2,6205	–0,22732	0,003277	0,98
10.17.2007	2,6115	–0,23142	–0,0041	1,23
10.18.2007	2,5813	–0,24559	–0,01417	4,25
10.19.2007	2,5891	–0,24187	0,003719	1,11
10.22.2007	2,5794	–0,2465	–0,00463	1,39
10.23.2007	2,5730	–0,24959	–0,00309	0,93
10.24.2007	2,5719	–0,25013	–0,00053	0,16
10.25.2007	2,5371	–0,26745	–0,01733	5,20
10.26.2007	2,5142	–0,27931	–0,01186	3,56

6 Extracted from the web page www.akcje.net, 5.12.2008.

Table 1. Continued

10.29.2007	2,5126	–0,28015	–0,00084	0,25
10.30.2007	2,5173	–0,27768	0,002468	0,74
10.31.2007	2,5080	–0,28258	–0,0049	1,47

Source: Author's own calculations.

Table 2. Costs of delta-hedging in November 2007 for every day changes in a zero-risk portfolio

Quotation date	USD/PLN curency rate (average)	Delta	Delta change during one day	Costs of delta-hedging for one put option on 100000 USD (in USD)
11.02.2007	2,5136	–0,27962	0,002956	0,89
11.05.2007	2,5179	–0,27737	0,002256	0,68
11.06.2007	2,4955	–0,28926	–0,01189	3,57
11.07.2007	2,4826	–0,29626	–0,007	2,10
11.08.2007	2,4692	–0,30365	–0,00739	2,22
11.09.2007	2,483	–0,29604	0,007611	2,28
11.12.2007	2,5051	–0,28412	0,011919	3,57
11.13.2007	2,4945	–0,28979	–0,00567	1,70
11.14.2007	2,4866	–0,29407	–0,00428	1,28
11.15.2007	2,4992	–0,28727	0,006806	2,04
11.16.2007	2,5119	–0,28052	0,006749	2,02
11.19.2007	2,5125	–0,2802	0,000316	0,09
11.20.2007	2,4821	–0,29653	–0,01633	4,90
11.21.2007	2,4869	–0,29391	0,00262	0,79
11.22.2007	2,4808	–0,29724	–0,00333	1,00
11.23.2007	2,4848	–0,29506	0,002188	0,66
11.26.2007	2,4837	–0,29566	–0,0006	0,18
11.27.2007	2,4812	–0,29702	–0,00137	0,41
11.28.2007	2,466	–0,30543	–0,00841	2,52
11.29.2007	2,4627	–0,30728	–0,00185	0,55
11.30.2007	2,4568	–0,3106	–0,00332	0,99

Source: Author's own calculations.

Table 3. Costs of delta-hedging in December 2007 for every day changes in a zero-risk portfolio

Quotation date	USD/PLN curency rate (average)	Delta	Delta change during one day	Costs of delta-hedging for one put option on 100000 USD (in USD)
12.03.2007	2,4613	–0,30806	0,002533	0,7599
12.04.2007	2,4482	–0,31548	–0,00741	2,2239
12.05.2007	2,4445	–0,31759	–0,00211	0,6345
12.06.2007	2,451	–0,31388	0,003709	1,1127
12.07.2007	2,4394	–0,32052	–0,00664	1,9917
12.10.2007	2,4234	–0,32982	–0,0093	2,7906
12.11.2007	2,4274	–0,32748	0,002341	0,7023
12.12.2007	2,4312	–0,32527	0,002215	0,6645
12.13.2007	2,4671	–0,30482	0,02045	6,135
12.14.2007	2,4983	–0,28775	0,017067	5,1201
12.17.2007	2,5215	–0,27549	0,012261	3,6783
12.18.2007	2,5111	–0,28094	–0,00545	1,6353
12.19.2007	2,5179	–0,27737	0,003573	1,0719
12.20.2007	2,5202	–0,27617	0,001201	0,3603
12.21.2007	2,5184	–0,27711	–0,00094	0,282
12.27.2007	2,4696	–0,30343	–0,02632	7,8957
12.28.2007	2,4465	–0,31645	–0,01302	3,9066

Source: Author's own calculations.

Table 4. Total costs of delta hedging for a put option on USD/PLN

Total costs of delta-hedging	For one put option on 100000 USD (in USD)	% of costs reduction in comparison to every day changes
For every day changes	175,92	–
For every 2 days changes	144,54	17.84
For every 3 days changes	144,30	17.97
For every 5 days changes	128,45	26.98
For every 10 days changes	101,55	42.27

Source: Author's own calculations.

Costs of delta-hedge modified every day are equal to 175,92 USD (see Table 4). They can be reduced thanks to making changes in the portfolio less often. As data depicted in Table 4 show, changes made every two or three days let reduce costs by about 18%, whereas changes made every 5 days by almost 27% and changes made every 10 days by about 42% in comparison to every day changes. It proves that increasing time intervals of modification of a risk-free portfolio lets reduce costs by substantial amount. Of course, it influences the efficiency of the strategy, however it is not analysed in this paper. The frequency of changes in the portfolio should be assessed depending on the future volatility of the underlying asset market. As it is emphasized by A. Gupta[7], for every derivative there is an optimal frequency of reconstructing a risk free portfolio and it depends on forecasts of the underlying asset market.

Changes made every few days are not a flexible method of hedging because it may happen that the difference between the present market portfolio and the risk-free portfolio is extremely high earlier than changes are planned. It is possible when the volatility suddenly changes dramatically, which is probable because this parameter is not constant. So, if a hedger assumed some level of underlying asset volatility and decided for the certain length of time intervals for some period of time, if volatility changed, the strategy should be reconstructed, which means that time intervals between portfolio changes should be modified to assure the maximum efficiency possible for this kind of the hedging strategy. The result of costs reduction can be also obtained when portfolio modifications are made according to delta or underlying asset changes, which seems to be more flexible. These methods are discussed beneath.

The Delta Tolerance Strategy

This method of hedging was suggested by Whalley and Wilmott[8]. It means changes in a hedging position when delta moves by the earlier stated value from the perfect position. Mathematically it is presented as:[9]

$$\tau_1 = t, \tau_{i+1} = \inf\left\{\tau_1 < \tau < T{:} \left| \Delta - \frac{\partial V}{\partial S} \right| > H\right\}, i = 1, 2\ldots,$$

7 See further: A. Gupta, *On neutral ground*, Risk, vol. 10, nr 7, 1997, p. 41.
8 A. E. Whalley, P. Wilmott, *Counting the Costs*, Risk nr 6, 1993, p. 59–66.
9 Ibidem.

where:

$\frac{\partial V}{\partial S}$ – the Black-Scholes hedge,

S – the underlying asset value,
V – the option value,
H – is a given constant tolerance defined by Whalley and Wilmott as:

$H = \frac{h}{S}$ (h = const.).

However, Zakamouline[10] states that simulations show that the strategy with a constant bandwidth H and the strategy with a bandtwidth given by $H = \frac{h}{S}$ have practically similar empirical performances.

Table 5. Total costs of the delta tolerance strategy for the examined option [USD]

	Costs of delta hedging with changes at				
	0.02	**0.03**	**0.04**	**0.05**	**0.06**
USD/PLN option	111.03	103.71	93.18	80.24	64.99
	% of costs reduction in comparison to every day changes				
	36.89	41.05	47.03	54.39	63.06

Source: Author's own calculations.

The tolerance value depends on the level of risk tolerance of a hedger. Calculations were conducted for the most typical values of H, that is 0.02; 0.03; 0.04; 0.05; 0.06. The higher the tolerance level is, the lower costs of hedging are. To be exact, costs of hedging with changes of delta by at least 0.02 are equal to 111.03 USD which is about 9% more than for every 10 days changes, 13% less than for every 5 days changes and 37% less than for every day changes (see Tables 4 and 5). When changes are made at delta moving by at least 0.03, costs of hedging can be reduced by 7% in comparison to changes at 0.02. However, it is still more than costs of delta hedge for every 10 days changes but 41% less than for every day changes.

10 V. I. Zakamouline, *Dynamic Hedging of Complex Option Positions with Transaction Costs*, Working Paper, Agder University College, February 2006, p. 8.

For changes at 0.05, costs are 80.24 USD that is 14% less than for 0.04, 26% less than for every 10 days changes and 54% less than for every day changes, whereas for modifications at 0.06 costs are 64.99 USD that is 23% lower than for 0.05, 56% lower than for every 10 days changes and 63% lower than for daily portfolio reconstructions.

Hedging to a Fixed Bandwidth

This method is similar to the delta tolerance strategy. The only difference is that changes in a hedging position are made only to the earlier stated boundary, not to achieve a risk-free portfolio again. The strategy was proposed by Martellini and Priaulet[11]. Mathematically it can be expressed as:[12]

$$\Delta = \frac{\partial V}{\partial S} \pm H$$

where:

$\frac{\partial V}{\partial S}$ – the Black-Scholes hedge,

V – option's price,

H – an earlier assumed constant which depends on the hedger's tolerance towards risk. The higher risk tolerance the higher value of H.

Changes to the boundary instead of creation of the risk free portfolio every time allow the hedger to reduce costs more than the delta tolerance strategy presented above.

The Underlying Asset Tolerance Strategy

The strategy, which was proposed by Henrotte, prescribes rehedging to the Black-Scholes delta when the percentage change in the value of the underlying asset exceeds the prescribed tolerance:[13]

11 See further: L. Martellini, P. Priaulet, *Competing Methods for Option Hedging in the Presence of Transaction Costs*, Journal of Derivatives 2002, 9(3), p. 26–38 and V. I. Zakamouline, *Dynamic Hedging*..., op. cit., p. 10.

12 V. I. Zakamouline, *Dynamic Hedging*..., op. cit., p. 10.

13 See: P. Henrotte, *Transaction Costs and Duplication Strategies*, Working Paper, Stanford University and HEC, 1993 and V. I. Zakamouline, *Dynamic Hedging*..., op. cit., p. 9.

$$\tau_1 = t, \tau_{i+1} = \inf\left\{\tau_1 < \tau < T: \left| \frac{S(\tau)-S(\tau_1)}{S(\tau_1)} \right| > h\right\}, i = 1,2,\ldots,$$

where:
h – a given constant percentage.

The higher is the risk tolerance, the higher is h.

Table 6. Total costs of the underlying asset tolerance strategy for the examined option [USD]

	Costs of delta hedging with changes by				
	1%	**2%**	**3%**	**4%**	**5%**
USD/PLN option	84.62	77.30	72.41	70.11	64.99
	% of costs reduction in comparison to every day changes				
	51.90	56.06	58.84	60.15	63.06

Source: Author's own calculations.

The analysis of the underlying asset tolerance strategy was done by the most typical values of the underlying asset change i.e. 1%,2%,3%,4% and 5%. Data depicted in Tables 4 and 6 show that costs of delta hedging modified at a 1% underlying instrument change are 84.62 USD which is 17% lower than for every 10 days changes and 52% lower than for every day modifications of the risk-free portfolio. If changes are made when the underlying asset moves by at least 2%, costs are equal to 77.30 USD that is 24% lower than for every 10 days changes and 56% lower than for daily portfolio reconstruction. For a 3% change costs are reduced by almost 59%, for a 4% change by 60%, whereas for a 5% change by 63%. It is worth emphasizing that the amount of costs reduction is the same for a 5% change of the underlying asset as for a 0.06 change of the delta. It would be interesting to examine the efficiency of the hedging strategy at these two levels. It will be the subject of the author's research in the nearest future.

Final Conclusions

The analysis shows that banks have a wide range of possibilities of reducing costs of hedging strategies. Choosing the correct method and its parameters should be preceded with deep examinations of both the underlying asset market and the whole economic situation. The important factor that must be taken into consideration is the risk tolerance of a bank.

Examinations were conducted on EUR/PLN options because these are one of the most frequently sold options on the Polish financial market. However conclusions could be applied to other kinds of options when they start to be widely traded in Poland.

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Data from the internet page: www.akcje.net