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Stochastic Experiments in Stabilisation of Money Market Benchmarks

Abstract

The main input of this research is a stochastic model of a theoretical panel of contributors (banks) to a money market index. The model proved to constitute a useful environment for testing various index formulae, their characteristics and some trade-offs that may arise while deciding on the particular benchmark's design. It may be also used to evaluate indices without historical data or stress them against different scenarios of adverse changes in market conditions or panellists' behaviour. The hypothetical problems with changes in the panel's composition as well as the irregularity of daily contributions may strongly influence the utility of a final benchmark to be used in medium and long term loan contracts, especially with retail clients. Our focus is on several selected classes of benchmarks' formulae that are derived from the raw index and allow for some confinement of the mentioned drawbacks while decreasing quality measured by other criteria (the goodness of fit). The set of classes include: the geometric time weights with different smoothing parameters and observation window's length used on the original raw index, stabilisation of the raw index in bands, rolling window volume weights re-balancing and finally the geometric time weights performed on log-volume transformed index. The potential trade-offs in such a benchmark's stabilisation efforts are shown.

Key words: financial market indices, interest rate benchmarks, compound Poisson process, index volatility reduction, transaction based benchmarks

JEL: G12, G17, E43

Eksperymenty stochastyczne w stabilizowaniu wskaźników referencyjnych rynku pieniężnego

Streszczenie

W artykule zaproponowano nowe podejście do badania wskaźników referencyjnych rynku pieniężnego w postaci modelu stochastycznego opisującego dynamikę panelu banków przekazujących informacje o transakcjach depozytowych do pewnego repozytorium lub agenta kalkulacyjnego.

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Model wykorzystano do przetestowania różnych klas i formuł matematycznych indeksów, zbadania ich własności oraz wskazania rozwiązań technicznych skutkujących zmniejszeniem ich zmienności. Środowisko to może być z powodzeniem zastosowane także to badania indeksów, co do których dane historyczne są mało dostępne lub nie istnieją. Potencjalne problemy wynikające ze zmian składu panelu a także z nieregularności dziennych kontrybucji danych panelistów do repozytorium istotnie wpływają na jakość tworzonego wskaźnika referencyjnego (benchmark-u), który może być używany w średnio- i długoterminowych kontraktach kredytowych (w szczególności zawieranych przez banki z klientami detalicznymi).

Artykuł zawiera klasyfikację takich formuł wyliczania wskaźników referencyjnych, które skutkują powstaniem wskaźnika o mniejszej zmienności niż dzienna średnia ważona wolumenem (indeks „surowy”). Zbiór rozważanych klas obejmuje: indeksy ważone geometrycznie względem czasu z różnymi parametrami wygładzającymi i różnymi szerokościami okna obserwacyjnego, indeksy stabilizowane w przedziałach, indeksy zależne od średnich wag w różnych okienkach czasowych oraz indeksy ważone geometrycznie względem czasu oparte o przekształcony logarymicznie indeks „surowy” (względem wolumenu transakcji depozytowych). W ostatniej części omówiono możliwe wybory między akceptowalnym poziomem jakości dopasowania nowego benchmarku do indeksu „surowego” a jego zmiennością.

Słowa kluczowe: wskaźniki referencyjne, rynek pieniężny, złożony proces Poissona, redukcja zmienności indeksu.

Introduction

At the present time, after so-called *LIBOR scandal* and its consequences, there is a great debate on new money market benchmarks design. A comprehensive overview of the scandal with a special focus on the manipulation techniques and their scale may be found in Duffie & Stein (2015, pp. 192–212) or Ghandi, Golez, Jackwerth & Plazzi (2015). Details of this historical discussion are out of scope of this research but it suffices to say that the key change of the paradigm proposed and broadly agreed upon is that money market benchmarks should be real transaction based rather than hypothetical questionnaire’s results averaging as it was and still is the case. This thought is reflected in IOSCO (2013, chapter: Quality of the Benchmark) where one may read that, “*The data used to construct a Benchmark should be based on prices, rates, indices or values that have been formed by the competitive forces of supply and demand and be anchored by observable transactions entered into at arm’s length between buyers and sellers in the market [...] the Benchmark measures*”.

Before the Benchmark reform (henceforth: BMR¹), the most popular money market indices and therefore benchmarks were usually calculated on the basis of a set of quotes given individually by a group of pre-agreed list of banks, called panel. The size of a particular panel might depend on specific regulations, but it was unusual to observe less than 8 and more than 30 banks in a panel. After the BMR reform,

¹ EU Regulation 2016/1011 of the European Parliament and of the Council of 8 June 2016 on indices used as benchmarks in financial instruments and financial contracts or to measure the performance of investment funds.

from qualitative view point, we may distinguish between the panel that contributes non-transactional information (hypothetical quotes) to calculate an index and the panel (probably much broader) of banks or other institutions which report their real transactions to the calculation agent in order to determine a transaction-based index. This research elaborates on the potential dilemmas one may have when establishing rules of an index in the latter case. Since the BMR regulations do not precisely define what means that than an index or benchmark is transaction-based, leaving the final decision to national regulators, we have some degrees of freedom in our exploratory analysis, i.e. dates of transactions taken into the consideration, system of weights, other mathematical transformations of the original data.

The article 11 of BMR contains the following provision on input data: “[data] *shall be sufficient to represent accurately and reliably the market or economic reality that the benchmark is intended to measure. The input data shall be transaction data, if available and appropriate.*” We believe that a simple volume weighted average rate calculated from contributed transactions from a certain day or period is the closest representation of what one may call a transaction-based index. For that reason, we treat this raw index as a benchmark in the horse-race of alternative index formulae later in this paper.

Even without precise new recipes ready to be implemented now from the regulatory point of view, it is possible to consider some practical aspects that may arise when dealing with a panel of banks contributing their real transactions to a repository and calculation agent. Adding to dilemmas of benchmarks’ reform elaborated by Mielus (2016) our main focus here is to propose a stochastic model of a panel and list some solutions to potential prohibitive volatility of volume weights in such benchmarks. We acknowledge that the proposed techniques are not the unique and complete solution to the excessive volatility problem and there exists others i.e.: longer rate adjustment periods.

The paper is organised as follows: in the first chapter we explain how to set-up a stochastic model of a panel of contributors and propose some nomenclature necessary in the subsequent parts of this article, the second chapter consists of a selection of important benchmark classes (from mathematical perspective) and their generic characterisation. The third chapter proposes some measures of volatility and tracking error which leads to two trade-off spaces we use in the following – the fourth and fifth chapters, which present the details of the simulations, concluded as well as a discussion on the results obtained. Finally, we conclude and suggest further research on that topic.

1. Stochastic set-up of a panel

Let’s assume throughout this article that there exists a repository of all transactions performed in the money market of a certain tenor (i.e. 3M), to which every bank $i \in N$ in a chosen panel \mathfrak{B} contributes its transactional deposit information

(rates $r_{i,j,t}$ and volumes $v_{i,j,t}$) on daily basis. We assume every bank may have $M_{i,t}$ transactions to report on a certain day t and $j \in \{1, \dots, M_{i,t}\}$ is a particular deal's counter in a day t of i -th panellist. Based on that information a hypothetical calculation agent works out the current benchmark value according to pre-agreed set of rules and broadcasts it publicly.

For the sake of simplicity, we use daily aggregated amounts of different banks as building blocks for a hypothetical index calculation, defined as follows:

$$R_{i,t} = \frac{\sum_{j=1}^{M_{i,t}} r_{i,j,t} v_{i,j,t}}{\sum_{j=1}^{M_{i,t}} v_{i,j,t}} \quad V_{i,t} = \sum_{j=1}^{M_{i,t}} v_{i,j,t}$$

Furthermore we assume that the first simplest choice benchmark (which we use as a reference and starting point) would be a volume daily weighted average of rates – raw index – defined as follows:

$$I_t^{raw} = \frac{\sum_{i=1}^N R_{i,t} V_{i,t}}{\sum_{i=1}^N V_{i,t}}$$

In asset markets this type of index is commonly referred to as a volume weighted average price (VWAP).

With the aim of properly modelling a certain panel's index behaviour, we may assume now that the weighted rate $R_{i,t}$ contributed on a day t by the i -th panellist and corresponding aggregated volume $V_{i,t}$ are both stochastic processes. We propose the following approach:

1. there exists a notional market rate *known* to each panellist who sets its deposits rates negotiation policy with reference to it. This market rate follows an arithmetical Brownian motion process with some mean μ_{mkt} and variance σ_{mkt} starting at $R_{mkt,0}$;
2. the above-mentioned policy (\forall_i) is reflected in spreads $s_{i,t}$ to the hypothetical market rate, which also follow arithmetical Brownian motion processes with means $\mu_{spr,i}$ and variances $\sigma_{spr,i}$ starting at $s_{i,0}$. We assume no correlation between any of the Brownian motions;
3. hence the weighted rate may be described as: $R_{i,t} = R_{mkt,t} + s_{i,t}$;
4. each aggregated volume is normally distributed with some $\mu_{vol,i}$ and variance $\sigma_{vol,i}$ or follows compound Poisson process (of normally distributed variables) with parameter λ_i^{*2} . For the sake of simplicity we define:

$$\lambda_i = \begin{cases} \lambda_i^* & \text{for compound Poisson volume processes} \\ 0 & \text{otherwise} \end{cases}$$

² Defined as number of days with nonzero reported volume to all days in a specified interval.

5. share of panellists with irregular volumes (compound Poisson) in the panel may be treated as a deep parameter of the model $\gamma = \gamma(\mathfrak{P}) \in [0,1]$.

In this approach a panel \mathfrak{P} on a market is described by set of parameters:

$$\Xi = \{N, \gamma, \mu_{mkt}, \sigma_{mkt}, R_{mkt,0}, (\mu_{spr,i})_{i=1}^N, (\sigma_{spr,i})_{i=1}^N, (s_{i,0})_{i=1}^N, (\mu_{vol,i})_{i=1}^N, (\sigma_{vol,i})_{i=1}^N, (\lambda_i)_{i=1}^N\}$$

If we now imagine that each of the parameters may be also drawn from some distributions (i.e uniform distributions over typical range a certain parameter is expected to be equal to) we may refer to such defined panel as a stochastic object (world) which we will use in the Monte Carlo experiments described later. Technically, we have to add two more parameters, namely: number of simulated panels S_p and number of paths simulated for each panel S_T , hence we propose the following nomenclature for a stochastic panel object: $\mathfrak{P}_{\Xi, S_p, S_T}$ and a stochastic panel's instance after k -th MC simulation³: $\mathfrak{P}_{\Xi, S_p, S_T; k}$.

Such characterised stochastic panel has a structure a reach enough to accommodate for some worlds that produce *excessively* volatile raw indices I_t^{raw} , which creates good grounds for testing alternative benchmarks' formulae. Volatility of a raw index may be high in this set-up due to:

1. high share (γ) of panellists with irregular volumes;
2. high variances of spreads ($\sigma_{spr,i}$) of the panellists with exceptionally high or low staring spreads and trends ($\mu_{spr,i}$);
3. high variances of volumes ($\sigma_{vol,i}$) of different panellists, especially the ones with unusually high or low spreads to the hypothetical market rate;
4. high hypothetical market rate variance (σ_{mkt}).

2. Benchmark's classes

In this section we list and assess several classes of money market benchmarks without an ambition of conducting exhaustive classification. These are examples of some possible approaches to index stabilisation.

2.1. Time weighted indices

The first class builds on the idea of a moving average of a fixed length but uses unequal time weights for different days inside the window. Usually, the fading monotonic weights are chosen, meaning that today's raw index has higher weight in the benchmark than the oldest in a window, which in turn, leads to better articulation of information ageing, where the latest information have substantially higher impact on current value of an index than the distant (the oldest) ones. This method obviously aims at benchmark's volatility reduction with some costs in tracking error measure on the other hand. Particular selection of weights with a certain class is a matter of choice in two-dimensional space (error measure vs volatility measure).

³ By *simulation* we mean here the set of paths simulated for all generated panels.

2.1.1. Arbitrary weights

One possibility is that the final user (beneficiary) or its agent chooses a time window K and a set of weights:

$$\mathcal{W} = \{w_0, w_1, \dots, w_{K-2}, w_{K-1} : w_0 \geq w_1 \geq \dots \geq w_{K-2} \geq w_{K-1} \wedge \sum_{d=0}^{K-1} w_d = 1\}$$

one thinks are appropriate for the usage in mind (i.e. $\mathcal{W}_{K=5} = \{0.3, 0.25, 0.2, 0.15, 0.1\}$, where the weight 0.3 corresponds to the most current observation).⁴ Benchmark formula of this class reads:

$$I_t^{arb}(\mathcal{W}_K) = \sum_{d=0}^{K-1} w_d I_{t-d}^{raw}$$

Since this class suffers from infinite many degrees of freedom it is useless in contributing to our research on trade-offs, but it leads to more compact class described below.

2.1.2. Geometric weights

We may want to choose smoothing parameter $0 < \alpha < 1$ and the window size K of our hypothetical benchmark to get the weights that are the result of a formula evaluation with just these two parameters. With this aim we set the weights equal to reversed geometric sequence and use a formula for the sum of finite geometric series to get:

$$I_t^{geo}(\alpha, K) = \sum_{d=0}^{K-1} \frac{\alpha(1-\alpha)^{d+1}}{1-(1-\alpha)^{K-1}} I_{t-d}^{raw}$$

This class is easily implementable for simulations and may be used in experiments when iterating over some space of smoothing parameter $\alpha \in \mathcal{A}$ and size of the window $K \in \mathcal{K}$. Some examples of the weights' structure depending on these two parameters are shown in Figure 1 in Appendix. In our experiments we used the following sets:

$$\begin{cases} \mathcal{A} = \{0.01, 0.05, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.7, 0.8, 0.9, 0.95\} \\ \mathcal{K} = \{5, 10, 20, 40, 50, 60\} \end{cases}$$

The lower α the smoother (flatter) weights it produces. As we will see the benchmarks with high values of α have similar characteristics to the original raw index they are derived from because the weights diminish rapidly within the given window as a day counter d increases.

⁴ Which was the case for the draft proposal of a benchmark derived from Polish money market repository SMRP during working meetings held in 2018.

2.2. Rolling window's average weight indices

Another class arises from the concept of stabilisation of weights used in the calculation of raw index in a day. As in the previous class we choose some window size K over which period we would like to stabilise volume weights. When on a certain day there is no data to report from a contributor we simply have to reweigh the scheme to include only the ones with nonzero contribution. It is sensible to choose $K \geq \frac{252}{\min_i \lambda_i^*}$ if the parameters λ_i^* express a fraction of expected occurrences of nonzero volume days in a business year consisting of 252 days. This condition's satisfaction would increase chances that at least one nonzero volume day of a certain contributor i occurred within the window frame and hence the effective weights are more stable. Impact of a volatility of volume is therefore reduced as well. Mathematical formula describing an index from this class follows:

$$I_t^{roll}(K) = \frac{\sum_{i=1}^N R_{i,t} (\sum_{s=t-K+1}^t V_{i,s})}{\sum_{i=1}^N \sum_{s=t-K+1}^t V_{i,s} \mathbf{1}_{\{V_{i,s} \neq 0\}}}$$

where $\mathbf{1}_{\{\cdot\}}$ is an indicator function.

2.3. Indices based on logarithmic transformation of volume

Next idea of reducing the impact of huge swings in volume and impact of a one-off massive transactions is to take logarithms⁵ of volumes before plugging them into raw index calculation:

$$I_t^{ln,raw} = \frac{\sum_{i=1}^N R_{i,t} \ln V_{i,t}}{\sum_{i=1}^N \ln V_{i,t}}$$

This trick yields in more equal treatment of every deal with less influence of transacted volume (i.e. transacted volumes of 1.000.000 and 100.000 translate approximately into weights of 0.5454 and 0.4545). When implementing this transformation on real or simulated data, one should mind the fact that if the volume traded falls into a band of $[0,1]$ one shall apply some modification (i.e. flat cut-off at) to avoid negative volume weights.

It is reasonable to mix that class with geometric weights, potentially creating even smoother and less volatile benchmarks:

$$I_t^{ln,geo}(\alpha, K) = \sum_{d=0}^{K-1} \frac{\alpha(1-\alpha)^{d+1}}{1-(1-\alpha)^{K-1}} I_{t-d}^{ln,raw}$$

⁵ Natural or decimal base logarithms would work equally efficient.

2.4. Crawling band indices

The last (but certainly not least) class we are considering is based on the concept of filtering the raw index within a given band width $2b$. The iteration algorithm is simple (\mathbb{T} – set of counters in the time series):

1. **start** $I_t^{band}(b) = I_t^{raw}$
2. **for all** $t + i \in \mathbb{T}$:
 - if:** $(I_{t+1}^{raw} > I_{t+i-1}^{band}(b) + b$ **or** $I_{t+1}^{raw} < I_{t+i-1}^{band}(b) - b)$ **then:** $I_{t+i}^{band}(b) = I_{t+i}^{raw}$,
 - else:** $I_{t+i}^{band}(b) = I_{t+i-1}^{band}(b)$

Because a band width choice is solely the final user's arbitrary decision we may argue that this kind of filter may be applied without any supervising authority, provided that the *underlying* raw index is a benchmark according to BMR regulations. Once crawling band class index is implemented we will have a piecewise constant benchmark, *visually* less volatile but if standard deviation is applied as a volatility measure it is easily verifiable that in fact it is on the contrary.

3. Measures of volatility and tracking error

The main assumption for further analysis and experiments is that the raw index calculated daily from volume weights is too volatile from a hypothetical user's perspective, be it a trader in a bank or a borrower with indexed loan to that raw benchmark. It is obvious that any stabilisation of a raw index (starting with simple moving averages) will decrease volatility of a new benchmark and increase its tracking error measure to the original raw index. In this section we define the spaces of these trade-offs.

Naturally, first choice of a volatility measure is a standard deviation, especially from financial derivatives traders' point of view. Indices that have very low standard deviation (basically fixed for a long time) tend not to attract attention of traders as they supposed to make money from the realised volatility. On the other hand, extreme and ephemeral spikes in standard deviation of an underlying instrument also bode ill for trading development, because of lack of homoscedasticity in the index process.

In our experiments we will use classical standard deviation (SD) measure calculated for the longest possible common calendar window for the whole group of alternative benchmarks we will be testing.

From the perspective of a borrower standard deviation is not the best measure of volatility it cares about. We may assume that the index of its choice would be the one that is semi-fixed in some longer than one day periods. That would not only increase predictability of financial costs in the first loan period for the borrower;

but also limit the feeling that index is a draw from a lottery, hence *random* and potentially questionable. We believe that one of the measures such as an index the user would consider is a mean absolute change (MAC) of a benchmark I_t^c as defined below:

$$MAC(I_s; [t, t + K]) = \frac{1}{K} \sum_{s=t}^{t+K-1} |I_{s+1} - I_s|$$

One of the possible *cost* measures of our benchmark's stabilisation efforts may be a mean absolute error (*tracking error*), formula for which is proposed below.

$$MAE(I_s; [t, t + K]) = \frac{1}{K + 1} \sum_{s=t}^{t+K} |I_s - I_s^{raw}|$$

The natural expectation is that the longer the period we are averaging over, the higher MAE of our index because it is not responding to much more volatile raw index, hence the absolute error cumulates. We follow findings of and use MAE as more natural and unambiguous measure of average error, skipping RMSE (root-mean square error).

We propose to compare the results of Monte Carlo simulations of different benchmarks' characteristics in two simple pairs: mean absolute error against standard deviation and mean absolute error against mean average change. We expect that the plots of average values of the measures used (MAE, SD, MAC) in these two paired spaces exhibit downward slope, hence allowing for an introduction of an optimal trade-off sets concept. An index belongs to that set if there is no better index in that space, were by *better* we mean the one with smaller volatility measure value and smaller tracking error measure than all the other indices in that particular space. Formal definition of the optimal set $\mathcal{O}_{\mathcal{J}, \mathfrak{P}}$ for a given list of tested indices \mathcal{J} and stochastic panel $\mathfrak{P}_{\Xi, S_p, S_T}$ is proposed below:

$$\mathcal{O}_{\mathcal{J}, \mathfrak{P}, MAE, SD} = \{I \in \mathcal{J}: \nexists I' \text{ s.t. } \overline{MAE}(I) > \overline{MAE}(I') \wedge \overline{SD}(I) > \overline{SD}(I')\}$$

and

$$\mathcal{O}_{\mathcal{J}, \mathfrak{P}, MAE, MAC} = \{I \in \mathcal{J}: \nexists I' \text{ s.t. } \overline{MAE}(I) > \overline{MAE}(I') \wedge \overline{MAC}(I) > \overline{MAC}(I')\}$$

where \overline{MAE} , \overline{MAC} , \overline{SD} are averages over their underlying values in S_p simulations of panel's $\mathfrak{P}_{\Xi, S_p, S_T}$ characteristics with S_T path simulations for each panel drawn.

4. Monte Carlo experiments set-up

In our experiments we have taken into consideration the following set of benchmarks \mathcal{J} from five classes we discussed in Chapter 2:

1. raw index RWA ⁶
2. arithmetical mean of contributed rates from a certain day AA
3. from arbitrary weights: $SMRPindx$ with $\mathcal{W} = \{0.3, 0.25, 0.2, 0.15, 0.1\}$
4. from geometric weights: 10 indices of a form $G_K:a$ with window sizes: $K \in \mathcal{K} = \{5, 5, 5, 5, 5, 10, 20, 40, 50, 60\}$ and smoothing parameters: $\alpha \in \mathcal{A} = \{0.9, 0.8, 0.7, 0.6, 0.01, 0.01, 0.01, 0.01, 0.01, 0.01\}$ respectively⁷
5. from mixture of geometric weights with logarithmic transformation of volumes: 6 indices of a form $L_K:a$ with window sizes: $K \in \mathcal{K} = \{5, 10, 20, 40, 50, 60\}$ and one smoothing parameter: $\alpha = 0.01$ respectively⁸
6. from rolling window's average weights: 6 indices of a form M_K with window sizes: $K \in \mathcal{K} = \{5, 10, 20, 40, 50, 60\}$ respectively⁹
7. from crawling band indices: 3 indices of a form S_b with half-band sizes $b \in B = \{0.0005, 0.001, 0.002\}$ ¹⁰
8. raw index on log-transformed volumes $RWAlog$.

As we wanted to perform simulations within reasonable time¹¹, we have chosen number of panels randomly generated from stochastic object $\mathfrak{P}_{\Xi, S_p, S_T}$ to be $S_p = 100$, with $S_T = 2500$ paths (one business year long – 252 timesteps per year) simulated for each panel.¹²

We have used two sets of meta-parameters Ξ_1 and Ξ_2 which deliberately differ from each other but the treatment of parameter λ responsible for the volume frequencies and indirectly for the share of irregular contributors in a panel. The set of common meta-parameters for both Ξ and their corresponding uniform distributions' parameters were:

1. number of contributors: $N \sim \mathcal{U}(5, 20)$
2. hypothetical market rate behaviour:
 $\mu_{mkt} \sim \mathcal{U}(-0.01, 0.01), \sigma_{mkt} \sim \mathcal{U}(0.001, 0.004), R_{mkt,0} \sim \mathcal{U}(-0.015, 0.1),$
3. contributors' spread to market behaviour:
 $(\mu_{spr,i})_{i=1}^N = [0], (\sigma_{spr,i})_{i=1}^N \sim \mathcal{U}(0.001, 0.008), (s_{i,0})_{i=1}^N \sim \mathcal{U}(-0.0035, 0.0035),$
4. contributors' volumes behaviour:
 $(\mu_{vol,i})_{i=1}^N \sim \mathcal{U}(500, 10000), (\sigma_{vol,i})_{i=1}^N \sim \mathcal{U}(200, 3000),$

⁶ As defined in point 1. Stochastic set-up of a panel.

⁷ Referred to as: G_5:0.9, G_5:0.8, G_5:0.7, G_5:0.6, G_5:0.01, G_10:0.01, G_20:0.01, G_40:0.01, G_50:0.01, G_60:0.01.

⁸ Referred to as: L_5:0.01, L_10:0.01, L_20:0.01, L_40:0.01, L_50:0.01, L_60:0.01.

⁹ Referred to as: M_5, M_10, M_20, M_40, M_50, M_60.

¹⁰ Referred to as: S_0.0005, S_0.001, S_0.002.

¹¹ Approximately 1 hour per stochastic panel on a standard Intel Core i7 machine.

¹² Implementation in Python with Numpy and Scipy modules.

where $\mathcal{U}(a, b)$ is a cdf function of uniform distribution in the range $[a, b]$. In the set \mathcal{E}_1 we used $(\lambda_i)_{i=1}^N \sim \mathcal{U}(52, 1095)$, which translates to $\approx 30\%$ share of irregular contributors and in the set \mathcal{E}_2 we took $(\lambda_i)_{i=1}^N = [0]$ to represent a stochastic panel with regular (daily) contributors only. The particular choice of meta-parameters' ranges used in uniform distributions above is driven by common sense and market behaviour hence it has no major impact on the results and findings in this research. Several other ranges have been tested, including, but not limited to, allowing for deeply negative rates (by setting negative trend and simulation's starting points of interest rates). Although, it is worth underlying that the higher the share of irregular contributions in a panel the more pronounced the effects described in the following chapter would become.

Results

The results of such set Monte Carlo experiments are listed in Table 1 and presented in Figures 2 to 5 in the Appendix. For the first list of meta-parameters we have the following optimal set in $MAE \times SD$:

$$\mathcal{O}_{J, \mathbb{R}_{\mathcal{E}_1, 100, 2500, j=1, MAE, SD}} = \{RWA, G_5: 0.9, G_5: 0.8, G_5: 0.7, G_5: 0.6, G_{10}: 0.01, G_{20}: 0.01, G_{40}: 0.01, G_{50}: 0.01, G_{60}: 0.01, L_{20}: 0.01, L_{60}: 0.01, M_5, M_{10}, M_{20}, M_{40}, M_{50}, M_{60}\}$$

Hence, we have 18 out of 29 tested indices in the optimal set constituting a trade-off space for choices between volatility and tracking error for the benchmark potential user and beneficiaries. The dominated indices here are:

$$\{AA, RWAlog, SMRPindx, G_5: 0.01, L_5: 0.01, L_{10}: 0.01, L_{40}: 0.01, L_{50}: 0.01, S_{0.0005}, S_{0.001}, S_{0.002}\}$$

Interestingly, the fact that crawling band indices seldom change does not translate into lower standard deviation, because quadratic function involved in its calculation is convex. Also majority of the smoothed log-volume weighted indices lay outside the optimal set. It is worth mentioning at this stage that the choice of smoothing parameters in geometric weights classes is intended to frugally include only the indices that lead to meaningful results. There was no point of including whole range of high α parameters into longer and longer windows because they produce pretty much the same results in that space. Extending window frame length for highly skewed (towards latest observation) time weights does not change dramatically the value of an index nor its volatility nor tracing error. Only much smoother weighting schemes (i.e.: $\alpha < 0.05$) differentiate the results when time windows are longer.

For the stochastic panel with no irregular contributors (\mathcal{E}_2) in the same space $MAE \times SD$ the optimal set is exactly the same although the position of the whole set is parallel shifted to the left (Figure 6 in the Appendix)].

In the $MAE \times MAC$ space the size of the optimal set is larger by 3–4 items, leaving behind only:

- $\{AA, RWAllog, SMRPindx, L_5: 0.01, L_{10}: 0.01, S_{0.001}, S_{0.002}\}$ for \mathcal{E}_1 and:
- $\{AA, RWAllog, SMRPindx, L_5: 0.01, L_{10}: 0.01, S_{0.0005}, S_{0.001}, S_{0.002}\}$ for \mathcal{E}_2 .

The comparison of the two optimal sets in this space is slightly different than in $MAE \times SD$. The indices with longer window size than 10 seem to produce very alike results, whereas smaller window indices show much higher differentiation.

Conclusions and further research

In general, greater window size results in some standard deviation's reduction in all contemplated indices, whereas mean average change is reduced much quicker, reaching an area in which further increase of K does not yield in volatility decrease but the error is growing faster. That area falls into $K \in [10, 20]$.

The indices based on log-volume transformed weights with geometric smoothing rarely belonged to optimal sets in our experiments, usually being dominated by some member of pure geometric weight indices with a longer window and the same smoothing parameter. It is worth mentioning that log-volume transformation always helped to reduce volatility measure values, but at a cost that forced these benchmarks outside an optimal trade-off sets.

The crawling band indices examined in the two trade-off spaces did not provide encouragement for their extensive usage, as they do not help to reduce standard deviation (in fact they increase it) and their help in reducing MAC is significant, but not enough to beat other indices from other classes.

The rolling window's average weight benchmarks proved to be promising, as they usually were members of our optimal sets beating arbitrary weight index (*SMRPindx*), but the increase in window size did not translate into major SD or MAC reductions.

The effective choice of benchmarks within the optimal trade-off sets depends on the perspective and the objectives of a final beneficiary i.e. trader in a bank hedging its funding costs, a retail mortgage borrower on a floating reference rate or even the monetary and regulatory authorities. We proposed flexible environment to test benchmark formulae in hypothetical panel's combinations. Using that set-up, we are able to tell if we have found optimal benchmark within contemplated list or not. Having the optimal trade-off, sets we may try to compare it with some *budget line* i.e. slope of cost to volatility trade-off which should yield in finding one benchmark given our preferences is optimal.

Further research may be also conducted when experimenting with correlation between Brownian motions in the stochastic panel model (between spreads and volume) as well as micro-modelling the transactions within one contributor's data. The real data from a deposit rate repository would also give rise to further calibration of stochastic panel model.

References

Duffie D. and Stein J.C. (2015), Reforming Libor and other financial market benchmarks, *Journal of Economic Perspectives*, 29, pp. 191–212.

EU Regulation 2016/1011 of the European Parliament and of the Council of 8 June 2016 on indices used as benchmarks in financial instruments and financial contracts or to measure the performance of investment funds (BMR).

Ghandi P., Golez B., Jackwerth J., and Plazzi A. (2015), Libor manipulation: Cui bono?, Available at <http://nbn-resolving.de/urn:nbn:de:bsz:352-268232>

Glasserman P. (2003), *Monte Carlo methods in financial engineering*, Springer.

Hirsa A. (2012), *Computational methods in finance*, CRC Press.

IOSCO (2013), Principles for financial benchmarks, final report.

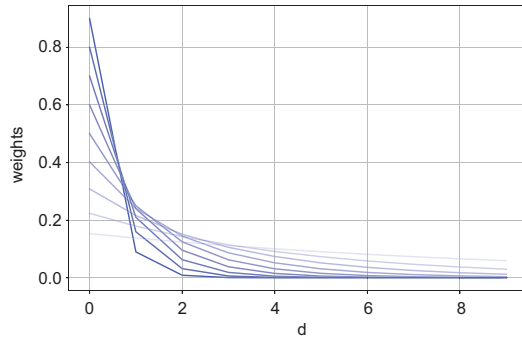
Mielus P. (2016), Financial market index reform dilemmas, *Gospodarka Narodowa*.

Willmott C.J. and Matsuura K. (2005), Advantages of the mean absolute error (MAE) over the root mean square error (RMSE) in assessing average model performance, *Climate research*, 30, pp. 79–82.

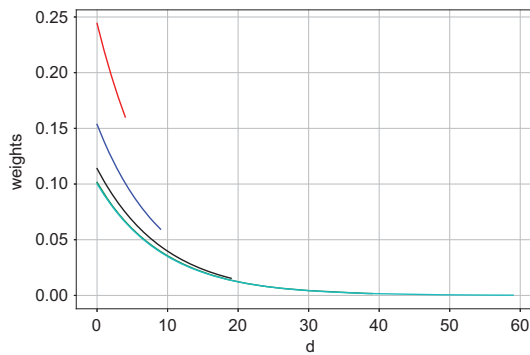
Appendix

Figure 1. Geometric weights' structure depending on parameter α and window size K

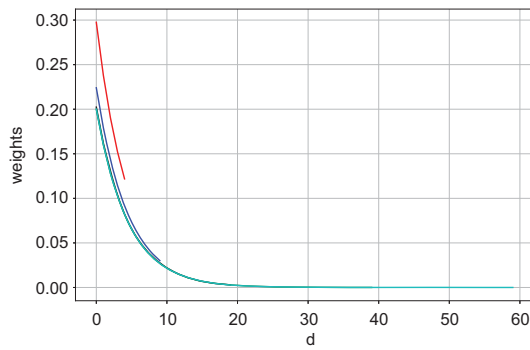
$K = 10, \mathcal{A} = \{0.9, 0.8, 0.7, 0.6, 0.5, 0.4, 0.3, 0.2, 0.1\}$



$\mathcal{K} = \{5, 10, 20, 40, 60\}, \alpha = 0.1$



$\mathcal{K} = \{5, 10, 20, 40, 60\}, \alpha = 0.2$



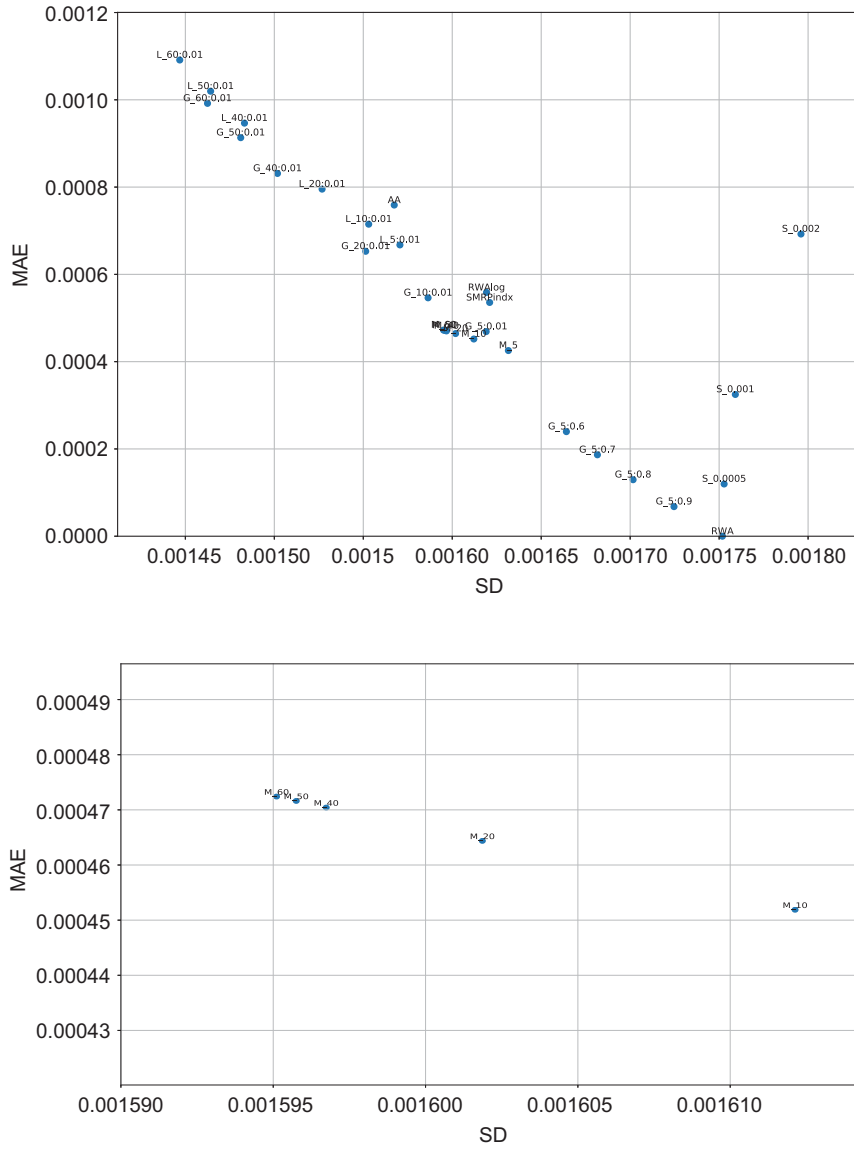
Note: lighter colours in the upper subplot correspond to lower values of α

Table 1. Results of Monte Carlo experiments with stochastic panels $\mathfrak{P}_{\varepsilon_1,100,2500}$ and $\mathfrak{P}_{\varepsilon_2,100,2500}$ for a set of indices \mathcal{J}

	$\mathfrak{P}_{\varepsilon_1,100,2500,i=1}$		$\mathfrak{P}_{\varepsilon_1,100,2500,i=2}$		$\mathfrak{P}_{\varepsilon_2,100,2500,i=1}$		$\mathfrak{P}_{\varepsilon_2,100,2500,i=2}$	
	SD	MAE	MAC	MAE	SD	MAE	MAC	MAE
AA	0.0015674	0.0007588	0.0001472	0.0007717	0.0014173	0.0006026	0.0001518	0.0006101
RWA	0.0017518	0.0000000	0.0006831	0.0000000	0.0015132	0.0000000	0.0004579	0.0000000
RWAllog	0.0016193	0.0005586	0.0003729	0.0005594	0.0014211	0.0005019	0.0001781	0.0005063
SMRPindx	0.0016210	0.0005356	0.0001756	0.0005298	0.0014434	0.0003801	0.0001276	0.0003790
G_5:0.9	0.0017246	0.0000678	0.0005872	0.0000674	0.0014991	0.0000449	0.0003961	0.0000445
G_5:0.8	0.0017016	0.0001295	0.0005033	0.0001287	0.0014870	0.0000864	0.0003411	0.0000857
G_5:0.7	0.0016816	0.0001865	0.0004281	0.0001854	0.0014762	0.0001254	0.0002916	0.0001244
G_5:0.6	0.0016641	0.0002396	0.0003608	0.0002381	0.0014668	0.0001621	0.0002472	0.0001610
G_5:0.01	0.0016191	0.0004689	0.0001573	0.0004645	0.0014422	0.0003282	0.0001160	0.0003271
G_10:0.01	0.0015864	0.0005462	0.0000884	0.0005373	0.0014200	0.0004023	0.0000702	0.0004038
G_20:0.01	0.0015513	0.0006529	0.0000535	0.0006354	0.0013902	0.0005113	0.0000462	0.0005182
G_40:0.01	0.0015018	0.0008312	0.0000360	0.0007981	0.0013425	0.0006893	0.0000335	0.0007078
G_50:0.01	0.0014811	0.0009132	0.0000326	0.0008727	0.0013218	0.0007686	0.0000309	0.0007930
G_60:0.01	0.0014625	0.0009918	0.0000304	0.0009440	0.0013028	0.0008433	0.0000292	0.0008737
L_5:0.01	0.0015706	0.0006675	0.0001002	0.0006669	0.0014061	0.0005499	0.0000726	0.0005570
L_10:0.01	0.0015530	0.0007150	0.0000627	0.0007101	0.0013929	0.0005905	0.0000523	0.0006012
L_20:0.01	0.0015268	0.0007953	0.0000425	0.0007817	0.0013686	0.0006655	0.0000391	0.0006834
L_40:0.01	0.0014832	0.0009465	0.0000315	0.0009158	0.0013252	0.0008059	0.0000308	0.0008378
L_50:0.01	0.0014642	0.0010197	0.0000293	0.0009807	0.0013057	0.0008726	0.0000290	0.0009115
L_60:0.01	0.0014468	0.0010912	0.0000277	0.0010442	0.0012878	0.0009372	0.0000277	0.0009829
M_5	0.0016316	0.0004254	0.0002155	0.0004253	0.0014550	0.0002738	0.0001836	0.0002704
M_10	0.0016121	0.0004519	0.0001744	0.0004519	0.0014461	0.0002893	0.0001640	0.0002857
M_20	0.0016019	0.0004644	0.0001607	0.0004644	0.0014414	0.0002962	0.0001581	0.0002925
M_40	0.0015967	0.0004704	0.0001567	0.0004703	0.0014391	0.0002995	0.0001565	0.0002956
M_50	0.0015958	0.0004716	0.0001562	0.0004715	0.0014387	0.0003001	0.0001563	0.0002962
M_60	0.0015951	0.0004724	0.0001559	0.0004723	0.0014384	0.0003005	0.0001562	0.0002966
S_0.0005	0.0017528	0.0001196	0.0005689	0.0001194	0.0015149	0.0001460	0.0003239	0.0001464
S_0.001	0.0017591	0.0003248	0.0003913	0.0003259	0.0015260	0.0003509	0.0001761	0.0003522
S_0.002	0.0017960	0.0006924	0.0001776	0.0006896	0.0015640	0.0006903	0.0000646	0.0006929

Source: Own elaboration

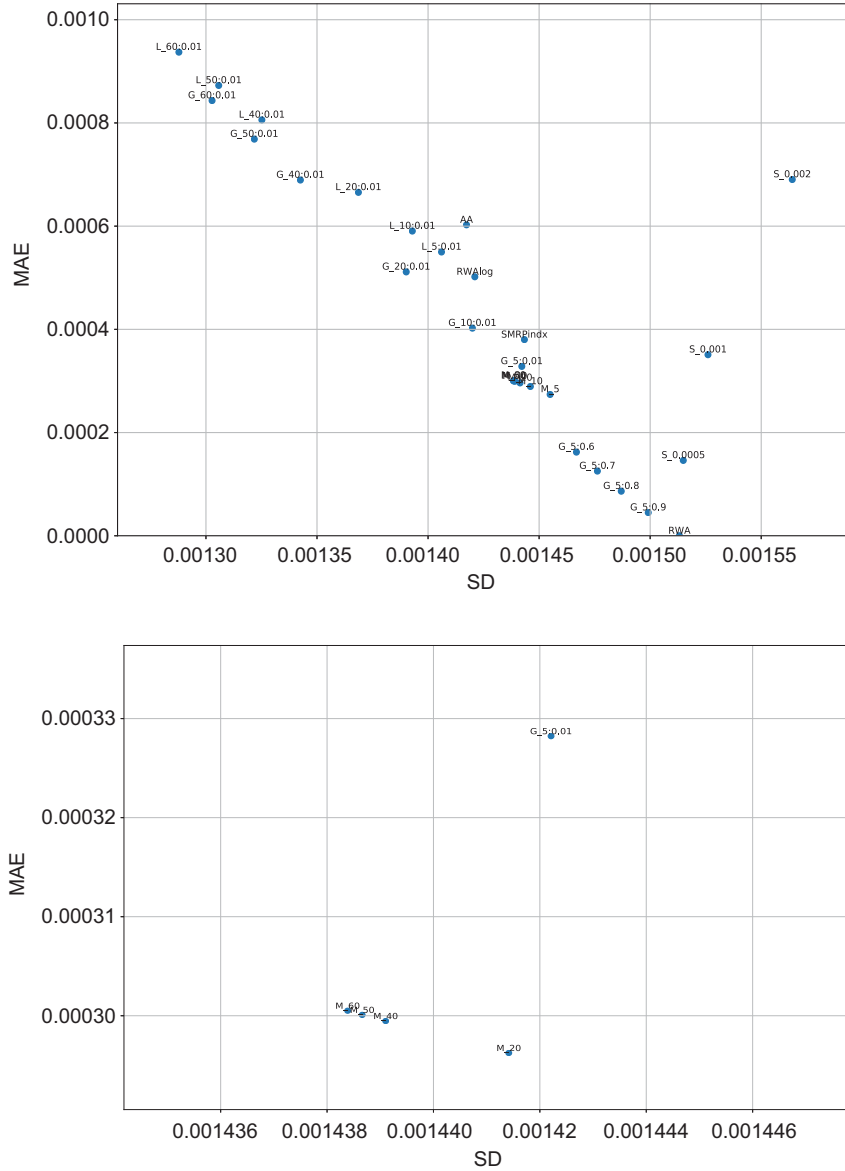
Figure 2. Trade-off space MAE × SD of stochastic panel $\mathfrak{P}_{\Xi_{1,100,2500},i=1}$



Note: The lower subplot represents a zoomed area of congestion on the upper pane.

Source: Own elaboration.

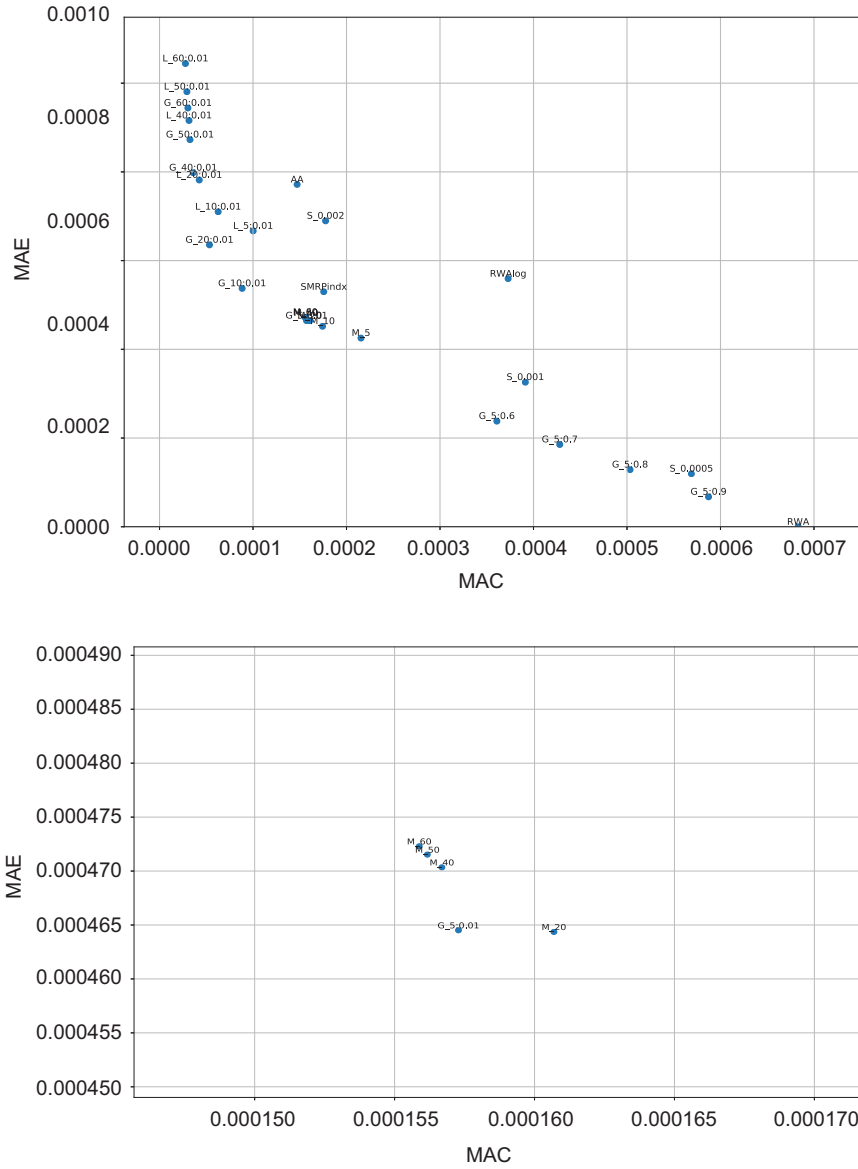
Figure 3. Trade-off space MAE × SD of stochastic panel $\mathfrak{P}_{\Sigma_2, 100, 2500, i=1}$



Note: The lower subplot represents a zoomed area of congestion on the upper pane.

Source: Own elaboration.

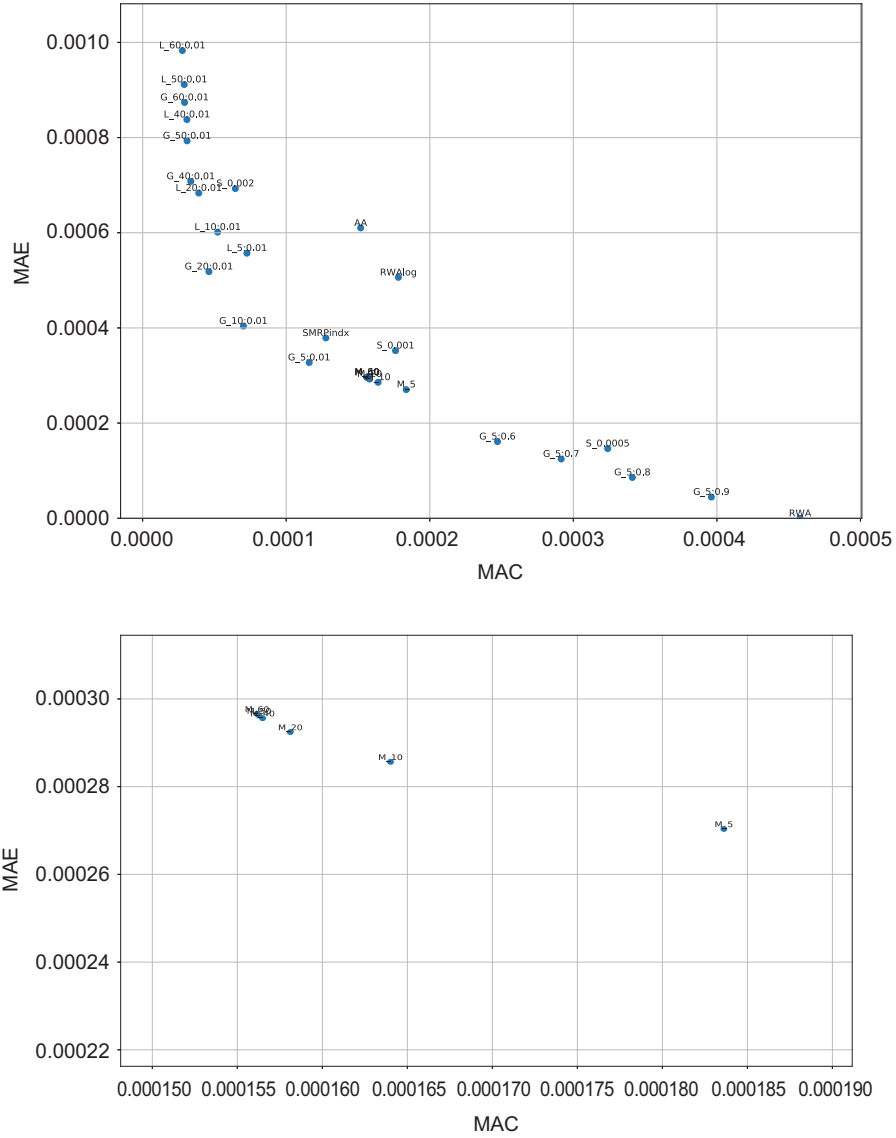
Figure 4. Trade-off space MAE × MAC of stochastic panel $\mathfrak{P}_{\Xi_{1,100,2500,i=2}}$



Note: The lower subplot represents a zoomed area of congestion on the upper pane.

Source: Own elaboration.

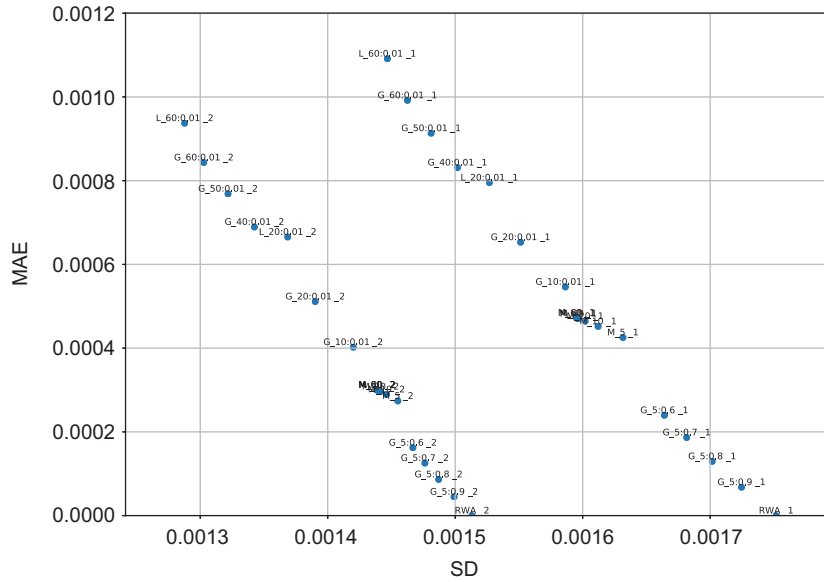
Figure 5. Trade-off space MAE × MAC of stochastic panel $\mathfrak{P}_{\mathcal{E}_2,100,2500,i=2}$



Note: The lower subplot represents a zoomed area of congestion on the upper pane.

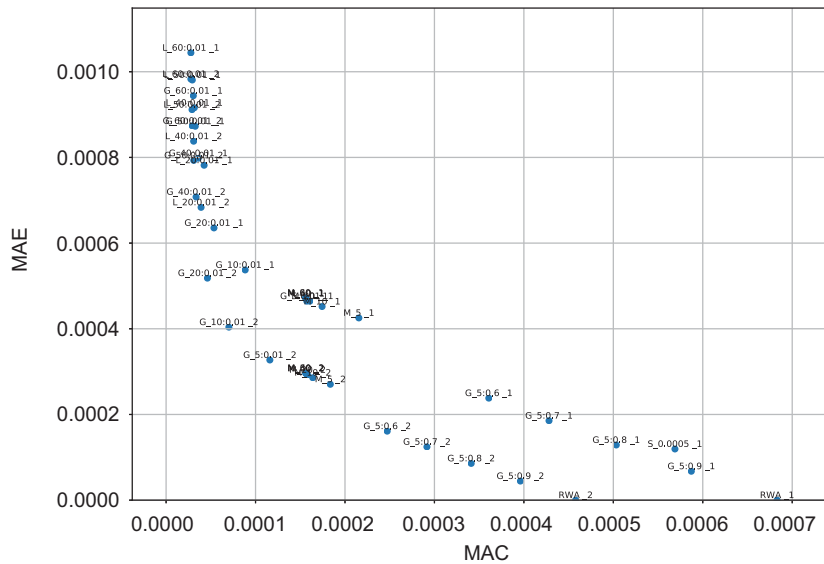
Source: Own elaboration.

Figure 6. Optimal sets compared in MAE × SD space



Source: Own elaboration.

Figure 7. Optimal sets compared in MAE × MAC space



Source: Own elaboration.